One Concrete Application of Point Set Theory in Measure Theory

Mariam Beriashvili
I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University
mariam_beriashvili@yahoo.com

We consider a modified version of the concept of measurability of sets and functions, and analyze this version from the point of view of additional set-theoretical axioms. The main feature of such an approach is that the measurability is treated not only with respect to a concrete given measure, but also with respect to various classes of measures. So, for a class M of measures, the measurability of sets and functions has the following three aspects:

a) absolute measurability with respect to M;
b) relative measurability with respect to M;
c) absolute non-measurability with respect to M.

With the aid of additional set theoretical axioms, we specify the above-mentioned aspects of measurability. It is also investigated how the classes of absolutely measurable, relatively measurable and absolutely non-measurable functions (with respect to a fixed class M of measures) behave under action of standard operations, such as composition, addition, multiplication, limit operation, and so on.

In particular, it is shown that:

(1) Any function, which has a λ_2 -massive graph, is relatively measurable with respect to the class of extensions of Lebsgue measure;
(2) There exists a Bernstein set which is absolutely negligible with respect to the class of all nonzero sigma-finite translation invariant measures on **R.**
(3) There exists a Bernstein set which is absolutely non-measurable with respect to the class of all nonzero sigma-finite translation invariant measures on **R.**

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