COMPACT SETS IN EUCLIDEAN SPACES AS AN IFS-ATTRACTORS

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Abstract:

An Iterated Function System (IFS) on the topological space X is the finite family $\mathcal{F}$ of contractive self-maps on X. Consider the space $\mathcal{H}(X)$ of nonempty, compact subsets of X. We can define a dynamical system $(\mathcal{H}(X), \mathcal{F})$ such that for each $K \in \mathcal{H}(X) \quad \mathcal{F}(K) = \bigcup_{f\in\mathcal{F}} f(K)$.

By the *attractor of Iterated Function System* or *IFS-attractor* we understand a nonempty compact set $A \subset X$ such that

$$A = \mathcal{F}(A) = \bigcup_{f\in\mathcal{F}} f(A)$$

and for every compact set $K \in \mathcal{H}(X)$ the sequence $\left(\mathcal{F}^n(K)\right)_{n=1}^{\infty}$ converges to A in the Vietoris topology on $\mathcal{H}(X)$.

We deal with the question of which compact metrizable spaces can be (homeomorphic to) attractors of Iterated Function Systems in the Euclidean space. We extend the result obtained by P. F. Duvall and L. S. Husch, and proved that given any compact metrizable finite-dimensional space X in Euclidean space which contains an open zero-dimensional uncountable subset, we can topologically transform the space X such that the image is the attractor of some IFS in $\mathbb{R}^n$. In other words it is enough to take a disjoint union of an arbitrary compact metrizable finite-dimensional space and some zero-dimensional uncountable space such that this union is the attractor of some IFS in Euclidean space.

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