Expected values for the a.d. number and 3D-iterations

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This is a part of a joint work with V. Fischer, S. Friedman and D. Montoya-Amaya

Some basic notions

- For $f, g \in \omega^\omega$, $f \leq^* g$ (*g dominates f*) means that $f(n) \leq g(n)$ for all but finitely many $n < \omega$.
- The *bounding number* $\mathfrak{b}$ is the least size of a $\leq^*$-unbounded subset of ω^ω.
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Fact

$\mathfrak{b} \leq \mathfrak{d}$ and $\mathfrak{b} \leq \mathfrak{a}$.

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Typically, it requires a lot of work to increase $\mathfrak{a}$ (beyond $\mathfrak{b}$).

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Conjecture

Many of these iterations force $\mathfrak{a} = \mathfrak{b}$... at least by technical changes in their construction.

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J. Brendle and V. Fischer succeeded to find the methods!

Theorem (Brendle and Fischer 2011)

If $\kappa \leq \mu$ are uncountable regular and $\mu^{\aleph_0} = \mu$ then there is a ccc poset forcing $\mathfrak{b} = \mathfrak{a} = \kappa \leq \mathfrak{s} = \mathfrak{c} = \mu$.

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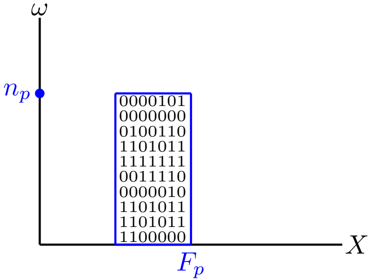
- **Conditions:** $p : F_p \times n_p \to 2$ where $F_p \in [X]^{<\aleph_0}$ and $n_p < \omega$.



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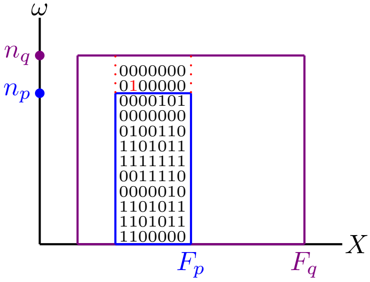
- **Order:** $q \leq p$ iff $p \subseteq q$ and, for any $i \in n_q \smallsetminus n_p$, there is at most one $x \in F_p$ such that $p(x, i) = 1$.



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The poset adds generically a family $\mathcal{A}|X := \langle A_x : x \in X \rangle$ of subsets of ω where

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- If $\mathcal{C}$ is a $\subseteq$-chain of sets and $Y = \bigcup \mathcal{C}$ then $\mathbb{H}_Y = \text{limdir}_{X \in \mathcal{C}} \mathbb{H}_X$. Therefore, if δ is a limit ordinal, $\mathbb{H}_\delta$ comes from the FS-iteration $\langle \mathbb{H}_\alpha, \dot{\mathbb{Q}}_\alpha \rangle_{\alpha<\delta}$ where $\dot{\mathbb{Q}}_\alpha$ is σ-centered.

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- $\mathbb{H}_X \simeq \mathbb{C}_{\omega_1}$ when $|X| = \aleph_1$.

Preservation properties

Definition (Brendle and Fischer 2011)

Let M be a transitive model of ZFC, $\mathcal{A} = \{A_z : z \in \Omega\} \in M$ a family of infinite subsets of ω and $B^* \in [\omega]^{\aleph_0}$.

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Lemma (Brendle and Fischer 2011)

Let $\mathcal{A}$, M and B^* as above. If B^* diagonalizes M outside $\mathcal{A}$ then $|X \cap B^*| = \aleph_0$ for any $X \in M \cap [\omega]^{\aleph_0} \smallsetminus \mathcal{I}(\mathcal{A})$.

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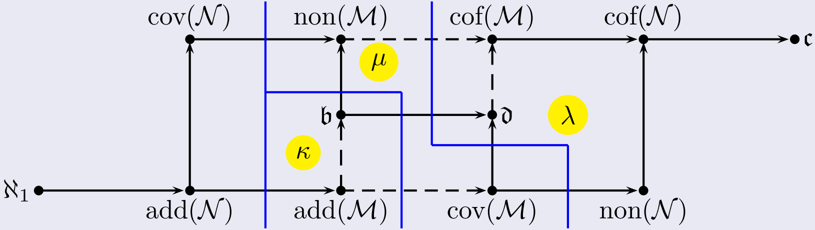
Lemma (Brendle and Fischer 2011)

In V, let Ω be a set and $z^* \in \Omega$. Then, in $V^{\mathbb{H}_\Omega}$, A_{z^*} diagonalizes $V^{\mathbb{H}_{\Omega \smallsetminus \{z^*\}}}$ outside $\mathcal{A}|(\Omega \smallsetminus \{z^*\})$.

An application

Theorem (essentially Brendle 1991)

Let $\kappa \leq \mu$ be uncountable regular cardinals, $\mu \leq \lambda$ such that $\lambda^{<\kappa} = \lambda$. Then, there is a ccc poset forcing



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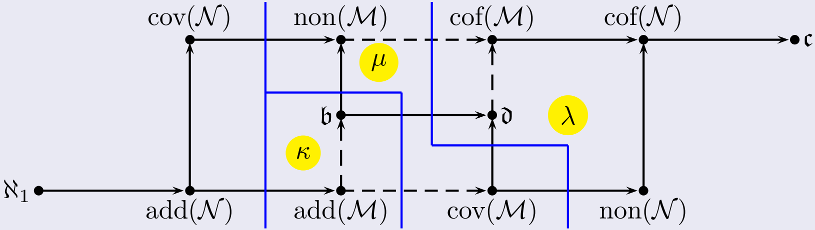

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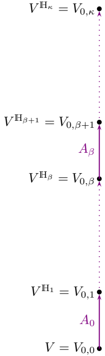

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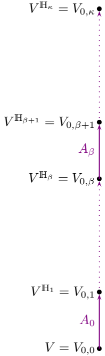

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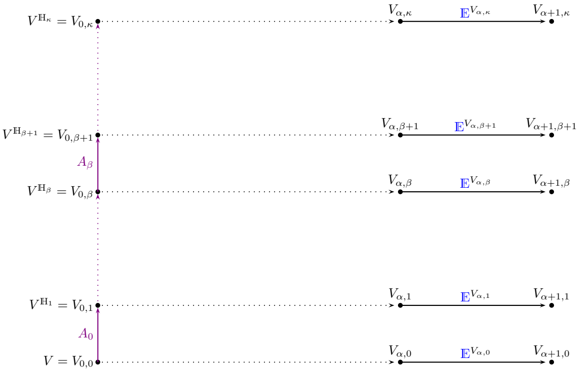

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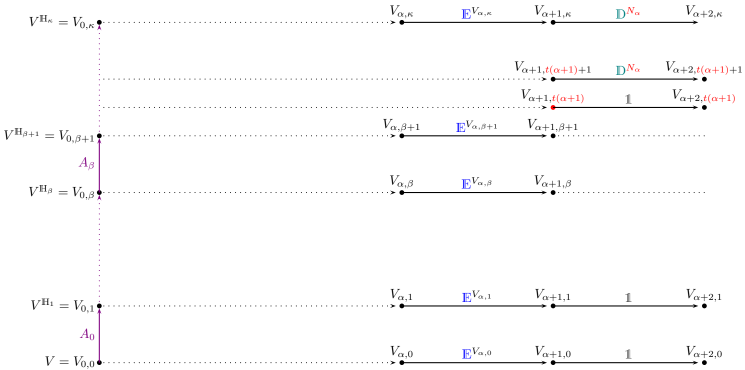

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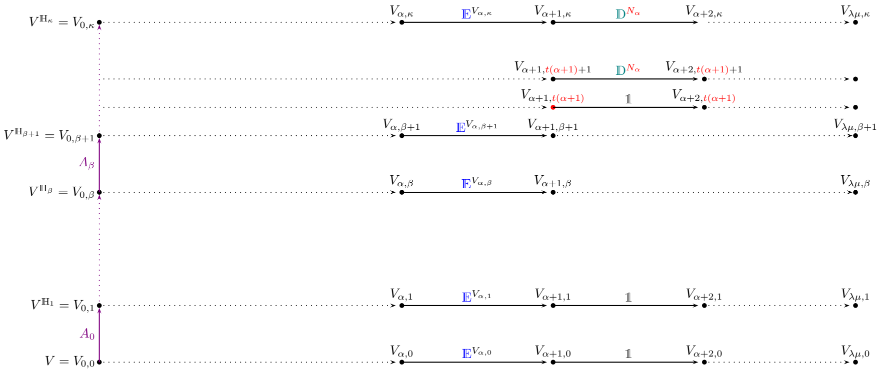

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Parallel FS-iterations of length $\lambda\mu$.

Technicalities

Fix $M \subseteq N$ transitive models of ZFC, $\mathcal{A} \in M$ and $B^* \in N$ diagonalizing M outside $\mathcal{A}$.

$B^* \in N$ •

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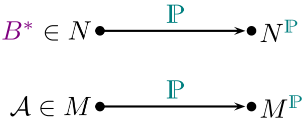

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Let $\mathbb{P} \in M$ be a poset. Then, in $N^{\mathbb{P}}$, B^* diagonalizes $M^{\mathbb{P}}$ outside $\mathcal{A}$.

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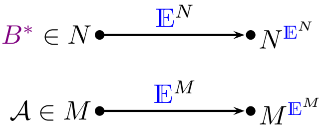

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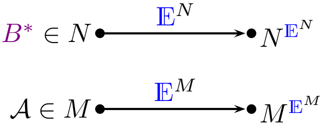

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The same holds for random forcing and Cohen forcing.

A general result

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Let κ be an uncountable regular cardinal. After forcing with $\mathbb{H}_\kappa$, any further FS-iteration where each iterand is either

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(ii) *a ccc poset of size $< \kappa$,*

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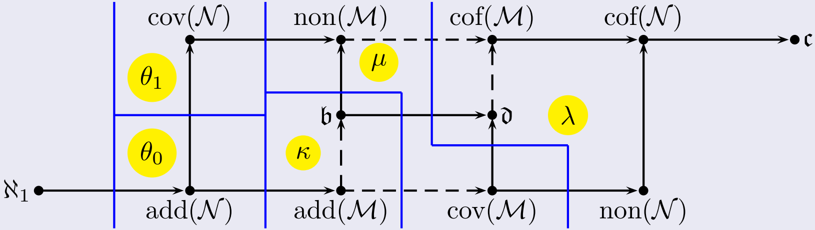
More examples

Corollary

Let $\theta_0 \leq \theta_1 \leq \kappa \leq \mu$ *be uncountable regular cardinals,* $\mu \leq \lambda$ *such that* $\lambda^{<\kappa} = \lambda$. *Then, there is a ccc poset forcing*



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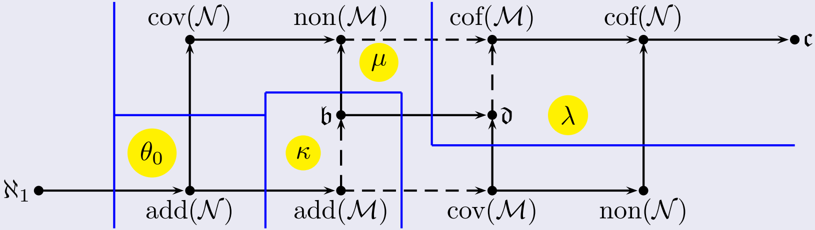

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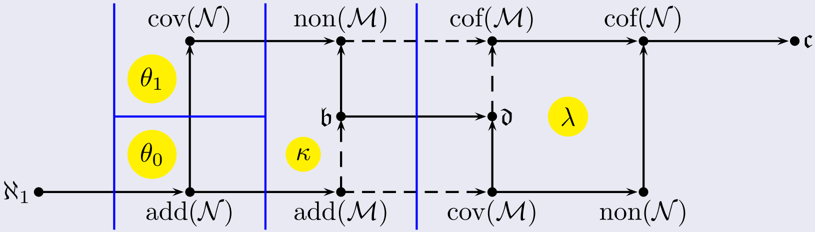

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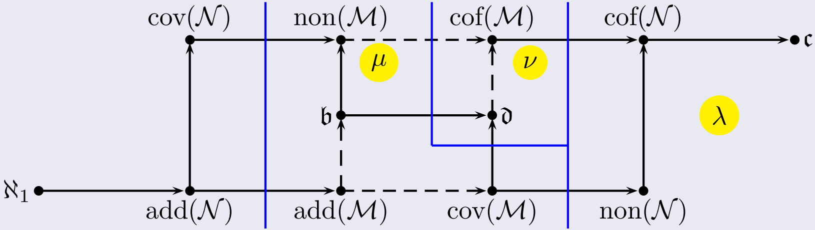
A matrix iteration application

Theorem (M. 2013)

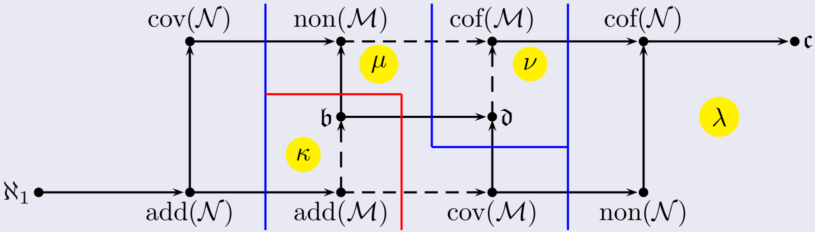
Let $\mu \leq \nu$ *be uncountable regular cardinals,* $\nu \leq \lambda$ *such that* $\lambda^{\aleph_0} = \lambda$. *Then, there is a ccc poset forcing*



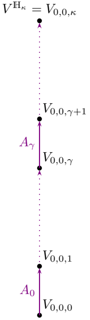
... turned into a 3D-iteration!

Theorem

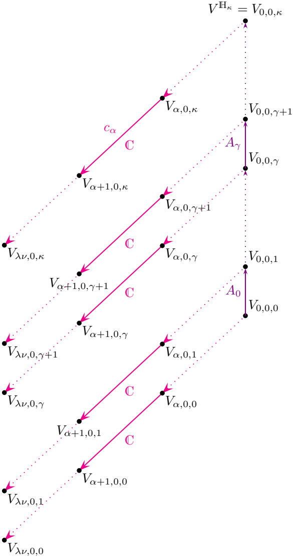
Let $\kappa \leq \mu \leq \nu$ *be uncountable regular cardinals,* $\nu \leq \lambda$ *such that* $\lambda^{\aleph_0} = \lambda$*. Then, there is a ccc poset forcing*



and $\mathfrak{a} = \mathfrak{b}$*.*



$V^{\mathbb{H}_\kappa} = V_{0,0,\kappa}$
$V_{0,0,\gamma+1}$
A_γ
$V_{0,0,\gamma}$
$V_{0,0,1}$
A_0
$V_{0,0,0}$


$V^{\mathbb{H}_\kappa} = V_{0,0,\kappa}$
$V_{\alpha,0,\kappa}$
$V_{0,0,\gamma+1}$
c_α
$\mathbb{C}$
A_γ
$V_{0,0,\gamma}$
$V_{\alpha+1,0,\kappa}$
$V_{\alpha,0,\gamma+1}$
$V_{\lambda\nu,0,\kappa}$
$V_{\alpha,0,\gamma}$
$V_{0,0,1}$
$V_{\alpha+1,0,\gamma+1}$
A_0
$V_{0,0,0}$
$V_{\alpha+1,0,\gamma}$
$V_{\lambda\nu,0,\gamma+1}$
$V_{\alpha,0,1}$
$V_{\lambda\nu,0,\gamma}$
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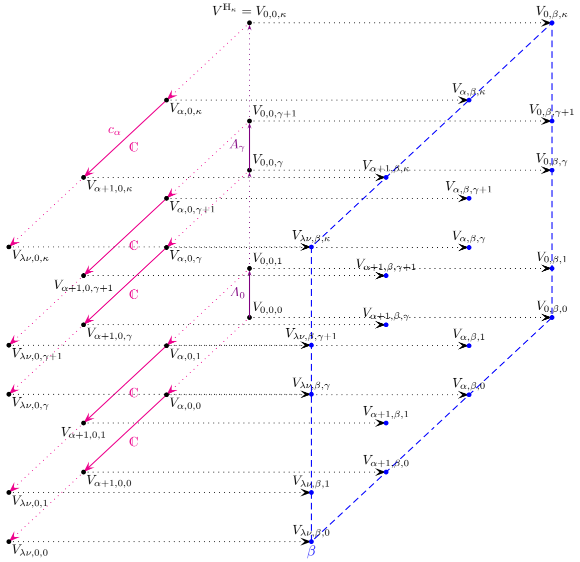
$V_{\alpha+1,0,0}$
$V_{\lambda\nu,0,1}$
$V_{\lambda\nu,0,0}$


$V^{\mathbb{H}_\kappa} = V_{0,0,\kappa}$
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$V_{0,0,\gamma+1}$
$V_{0,\beta,\gamma+1}$
c_α
$\mathbb{C}$
A_γ
$V_{0,0,\gamma}$
$V_{\alpha+1,\beta,\kappa}$
$V_{0,\beta,\gamma}$
$V_{\alpha+1,0,\kappa}$
$V_{\alpha,\beta,\gamma+1}$
$V_{\alpha,0,\gamma+1}$
$V_{\lambda\nu,\beta,\kappa}$
$V_{\alpha,\beta,\gamma}$
$V_{\lambda\nu,0,\kappa}$
$V_{\alpha,0,\gamma}$
$V_{0,0,1}$
$V_{\alpha+1,\beta,\gamma+1}$
$V_{0,\beta,1}$
$V_{\alpha+1,0,\gamma+1}$
A_0
$V_{0,0,0}$
$V_{\alpha+1,\beta,\gamma}$
$V_{0,\beta,0}$
$V_{\alpha+1,0,\gamma}$
$V_{\lambda\nu,\beta,\gamma+1}$
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$V_{\lambda\nu,0,\gamma+1}$
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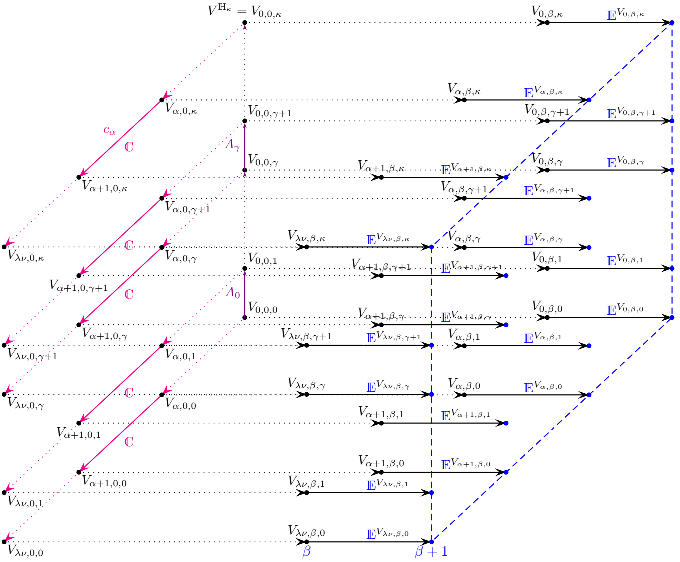
$V_{\lambda\nu,\beta,0}$
$V_{\lambda\nu,0,0}$
β



$V^{\mathbb{H}_\kappa} = V_{0,0,\kappa}$
$V_{0,\beta,\kappa}$
$\mathbb{E}^{V_{0,\beta,\kappa}}$
$V_{\alpha,\beta,\kappa}$
$\mathbb{E}^{V_{\alpha,\beta,\kappa}}$
$V_{\alpha,0,\kappa}$
$V_{0,0,\gamma+1}$
$V_{0,\beta,\gamma+1}$
$\mathbb{E}^{V_{0,\beta,\gamma+1}}$
c_α
$\mathbb{C}$
A_γ
$V_{0,0,\gamma}$
$V_{\alpha+1,\beta,\kappa}$
$\mathbb{E}^{V_{\alpha+1,\beta,\kappa}}$
$V_{0,\beta,\gamma}$
$\mathbb{E}^{V_{0,\beta,\gamma}}$
$V_{\alpha+1,0,\kappa}$
$V_{\alpha,\beta,\gamma+1}$
$\mathbb{E}^{V_{\alpha,\beta,\gamma+1}}$
$V_{\alpha,0,\gamma+1}$
$V_{\lambda\nu,\beta,\kappa}$
$\mathbb{E}^{V_{\lambda\nu,\beta,\kappa}}$
$V_{\alpha,\beta,\gamma}$
$\mathbb{E}^{V_{\alpha,\beta,\gamma}}$
$V_{\lambda\nu,0,\kappa}$
$V_{\alpha,0,\gamma}$
$V_{0,0,1}$
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$\mathbb{E}^{V_{\alpha+1,\beta,\gamma+1}}$
$V_{0,\beta,1}$
$\mathbb{E}^{V_{0,\beta,1}}$
$V_{\alpha+1,0,\gamma+1}$
A_0
$V_{0,0,0}$
$V_{\alpha+1,\beta,\gamma}$
$\mathbb{E}^{V_{\alpha+1,\beta,\gamma}}$
$V_{0,\beta,0}$
$\mathbb{E}^{V_{0,\beta,0}}$
$V_{\alpha+1,0,\gamma}$
$V_{\lambda\nu,\beta,\gamma+1}$
$\mathbb{E}^{V_{\lambda\nu,\beta,\gamma+1}}$
$V_{\alpha,\beta,1}$
$\mathbb{E}^{V_{\alpha,\beta,1}}$
$V_{\lambda\nu,0,\gamma+1}$
$V_{\alpha,0,1}$
$V_{\lambda\nu,\beta,\gamma}$
$\mathbb{E}^{V_{\lambda\nu,\beta,\gamma}}$
$V_{\alpha,\beta,0}$
$\mathbb{E}^{V_{\alpha,\beta,0}}$
$V_{\lambda\nu,0,\gamma}$
$V_{\alpha,0,0}$
$V_{\alpha+1,\beta,1}$
$\mathbb{E}^{V_{\alpha+1,\beta,1}}$
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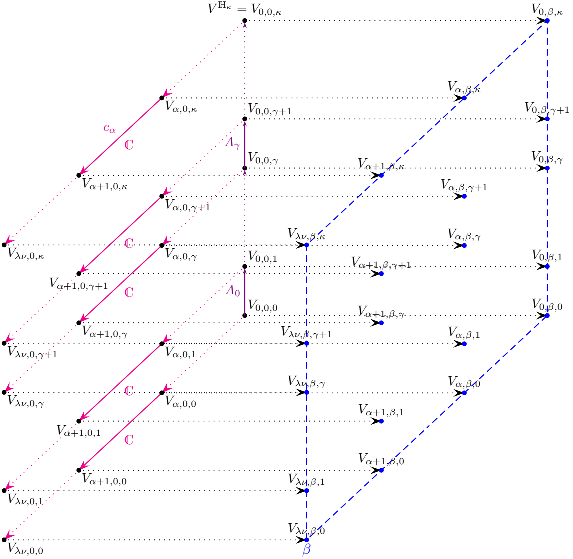
$V_{\alpha+1,\beta,0}$
$\mathbb{E}^{V_{\alpha+1,\beta,0}}$
$V_{\alpha+1,0,0}$
$V_{\lambda\nu,\beta,1}$
$\mathbb{E}^{V_{\lambda\nu,\beta,1}}$
$V_{\lambda\nu,0,1}$
$V_{\lambda\nu,\beta,0}$
$\mathbb{E}^{V_{\lambda\nu,\beta,0}}$
$V_{\lambda\nu,0,0}$
β
$\beta+1$



$V^{\mathbb{H}_\kappa} = V_{0,0,\kappa}$
$V_{0,\beta,\kappa}$
$V_{\alpha,\beta,\kappa}$
$V_{\alpha,0,\kappa}$
$V_{0,0,\gamma+1}$
$V_{0,\beta,\gamma+1}$
c_α
$\mathbb{C}$
A_γ
$V_{0,0,\gamma}$
$V_{\alpha+1,\beta,\kappa}$
$V_{0,\beta,\gamma}$
$V_{\alpha+1,0,\kappa}$
$V_{\alpha,\beta,\gamma+1}$
$V_{\alpha,0,\gamma+1}$
$V_{\lambda\nu,\beta,\kappa}$
$V_{\alpha,\beta,\gamma}$
$V_{\lambda\nu,0,\kappa}$
$\mathbb{C}$
$V_{\alpha,0,\gamma}$
$V_{0,0,1}$
$V_{\alpha+1,\beta,\gamma+1}$
$V_{0,\beta,1}$
$V_{\alpha+1,0,\gamma+1}$
$\mathbb{C}$
A_0
$V_{0,0,0}$
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$V_{\lambda\nu,\beta,\gamma+1}$
$V_{\alpha,\beta,1}$
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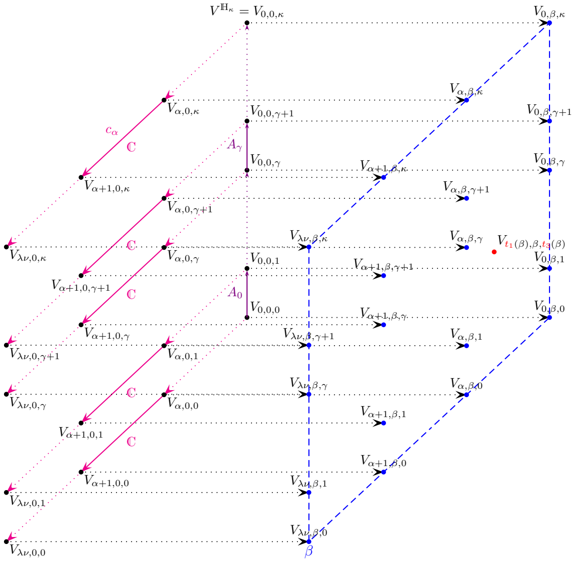
$V_{\alpha,0,1}$
$V_{\lambda\nu,\beta,\gamma}$
$V_{\alpha,\beta,0}$
$\mathbb{C}$
$V_{\lambda\nu,0,\gamma}$
$V_{\alpha,0,0}$
$V_{\alpha+1,\beta,1}$
$V_{\alpha+1,0,1}$
$\mathbb{C}$
$V_{\alpha+1,\beta,0}$
$V_{\alpha+1,0,0}$
$V_{\lambda\nu,\beta,1}$
$V_{\lambda\nu,0,1}$
$V_{\lambda\nu,\beta,0}$
$V_{\lambda\nu,0,0}$
β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$V_{\alpha,0,\kappa}$
$V_{0,0,\gamma+1}$
$V_{0,\beta,\gamma+1}$
c_α
$\mathbb{C}$
A_γ
$V_{0,0,\gamma}$
$V_{\alpha+1,\beta,\kappa}$
$V_{0,\beta,\gamma}$
$V_{\alpha+1,0,\kappa}$
$V_{\alpha,\beta,\gamma+1}$
$V_{\alpha,0,\gamma+1}$
$V_{\lambda\nu,\beta,\kappa}$
$V_{\alpha,\beta,\gamma}$
$V_{t_1(\beta),\beta,t_3(\beta)}$
$V_{\lambda\nu,0,\kappa}$
$V_{\alpha,0,\gamma}$
$V_{0,0,1}$
$V_{\alpha+1,\beta,\gamma+1}$
$V_{0,\beta,1}$
$V_{\alpha+1,0,\gamma+1}$
A_0
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$V_{\alpha+1,0,0}$
$V_{\lambda\nu,\beta,1}$
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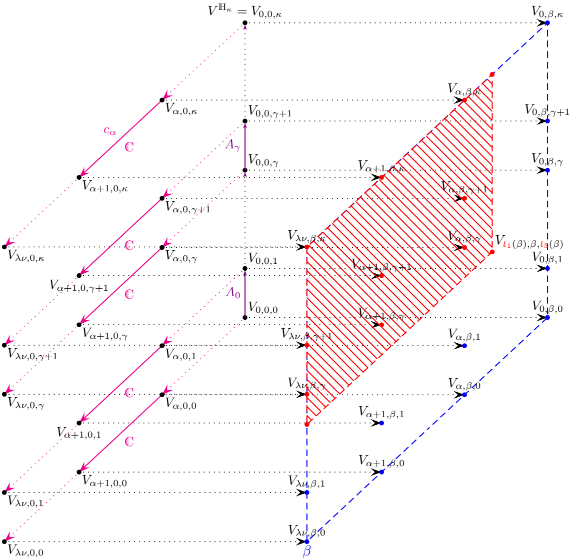
$V_{\lambda\nu,\beta,0}$
$V_{\lambda\nu,0,0}$
β



$V^{\mathbb{H}_\kappa} = V_{0,0,\kappa}$
$V_{0,\beta,\kappa}$
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$V_{0,\beta,\gamma+1}$
c_α
$\mathbb{C}$
A_γ
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$V_{0,\beta,\gamma}$
$V_{\alpha+1,\beta,\kappa}$
$V_{\alpha+1,0,\kappa}$
$V_{\alpha,\beta,\gamma+1}$
$V_{\alpha,0,\gamma+1}$
$V_{\lambda\nu,\beta,\kappa}$
$V_{\alpha,\beta,\gamma}$
$V_{t_1(\beta),\beta,t_3(\beta)}$
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$V_{0,\beta,1}$
$V_{\alpha+1,\beta,\gamma+1}$
$V_{\alpha+1,0,\gamma+1}$
A_0
$V_{0,0,0}$
$V_{0,\beta,0}$
$V_{\alpha+1,\beta,\gamma}$
$V_{\alpha+1,0,\gamma}$
$V_{\lambda\nu,\beta,\gamma+1}$
$V_{\alpha,\beta,1}$
$V_{\lambda\nu,0,\gamma+1}$
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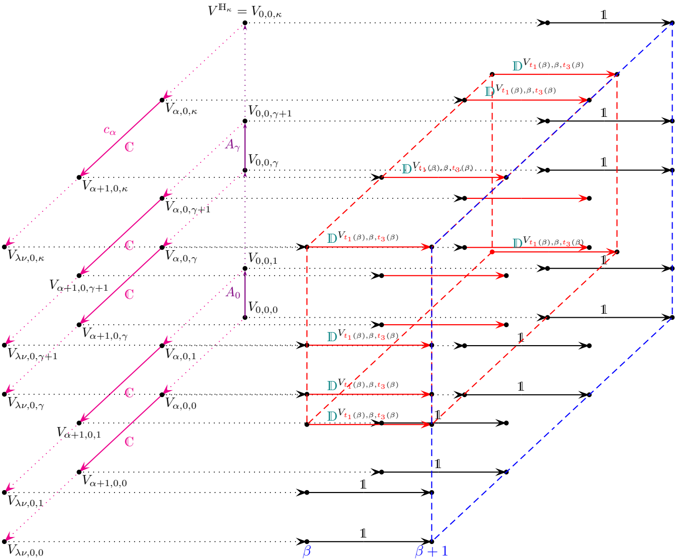
$V_{\lambda\nu,\beta,\gamma}$
$V_{\alpha,\beta,0}$
$V_{\lambda\nu,0,\gamma}$
$V_{\alpha,0,0}$
$V_{\alpha+1,\beta,1}$
$V_{\alpha+1,0,1}$
$V_{\alpha+1,\beta,0}$
$V_{\alpha+1,0,0}$
$V_{\lambda\nu,\beta,1}$
$V_{\lambda\nu,0,1}$
$V_{\lambda\nu,\beta,0}$
β
$V_{\lambda\nu,0,0}$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c_α
$\mathbb{C}$
A_γ
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$V_{\lambda\nu,0,\kappa}$
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$V_{\alpha+1,0,\gamma+1}$
A_0
$V_{0,0,0}$
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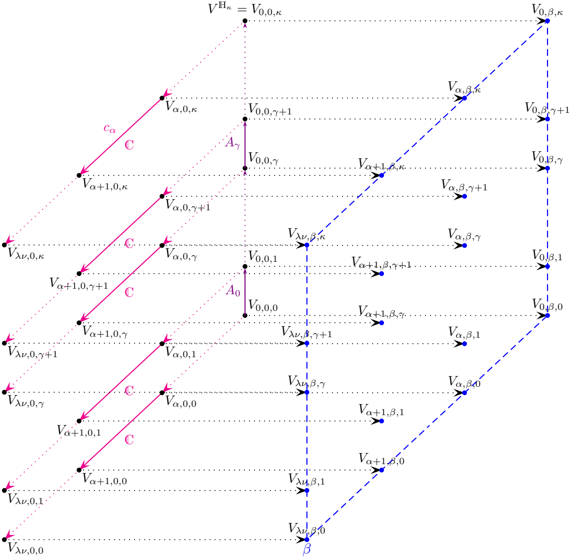
$V_{\lambda\nu,0,\gamma}$
$V_{\alpha,0,0}$
$V_{\alpha+1,0,1}$
$V_{\alpha+1,0,0}$
$V_{\lambda\nu,0,1}$
$V_{\lambda\nu,0,0}$
$\mathbb{D}^{V_{t_1(\beta),\beta,t_3(\beta)}}$
$\mathbb{1}$
β
$\beta+1$



$V^{\mathbb{H}_\kappa} = V_{0,0,\kappa}$
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$V_{0,\beta,\gamma+1}$
c_α
$\mathbb{C}$
A_γ
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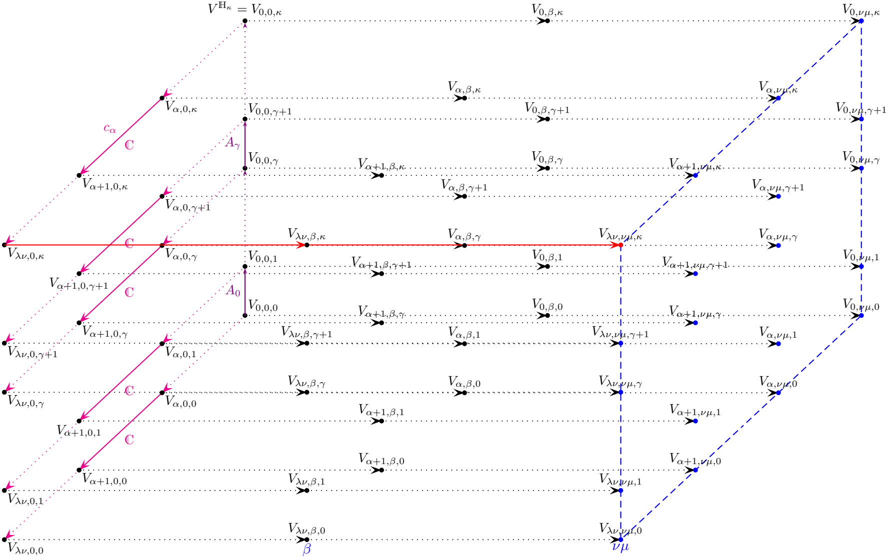
$V_{\lambda\nu,0,0}$
β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$V_{\alpha,0,\kappa}$
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$V_{0,\beta,\gamma+1}$
$V_{0,\nu\mu,\gamma+1}$
c_α
$\mathbb{C}$
A_γ
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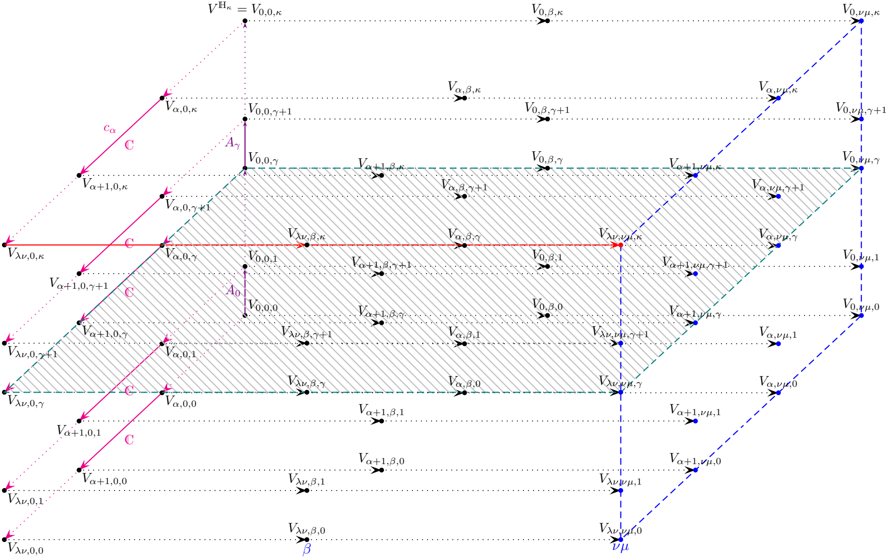
β
$\nu\mu$



$V^{\mathbb{H}_\kappa} = V_{0,0,\kappa}$
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$V_{0,\nu\mu,\kappa}$
$V_{\alpha,\beta,\kappa}$
$V_{\alpha,\nu\mu,\kappa}$
$V_{\alpha,0,\kappa}$
$V_{0,0,\gamma+1}$
$V_{0,\beta,\gamma+1}$
$V_{0,\nu\mu,\gamma+1}$
c_α
$\mathbb{C}$
A_γ
$V_{0,0,\gamma}$
$V_{\alpha+1,\beta,\kappa}$
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$V_{\alpha+1,\nu\mu,\kappa}$
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$V_{0,\nu\mu,1}$
$V_{\alpha+1,0,\gamma+1}$
A_0
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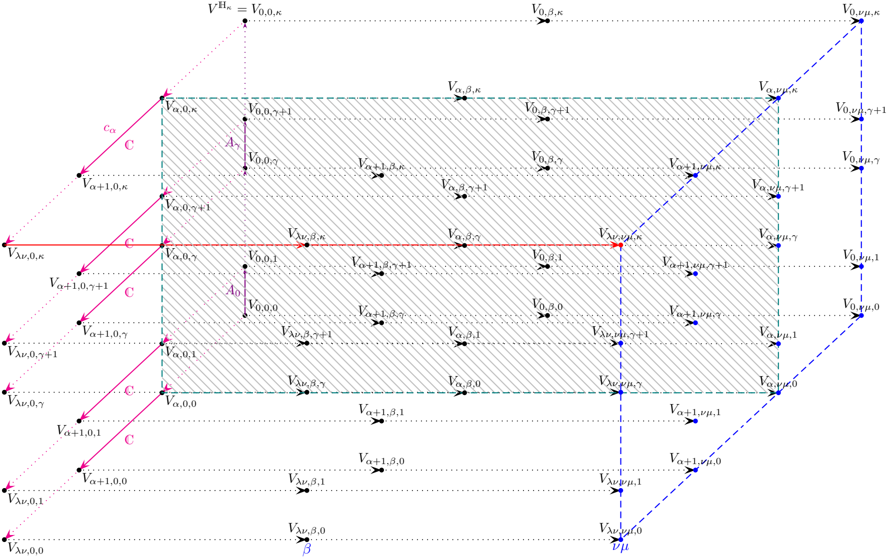
β
$\nu\mu$



$V^{\mathbb{H}_\kappa} = V_{0,0,\kappa}$
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$V_{0,\nu\mu,\kappa}$
$V_{\alpha,\beta,\kappa}$
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$V_{\alpha,0,\kappa}$
$V_{0,0,\gamma+1}$
$V_{0,\beta,\gamma+1}$
$V_{0,\nu\mu,\gamma+1}$
c_α
$\mathbb{C}$
A_γ
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A_0
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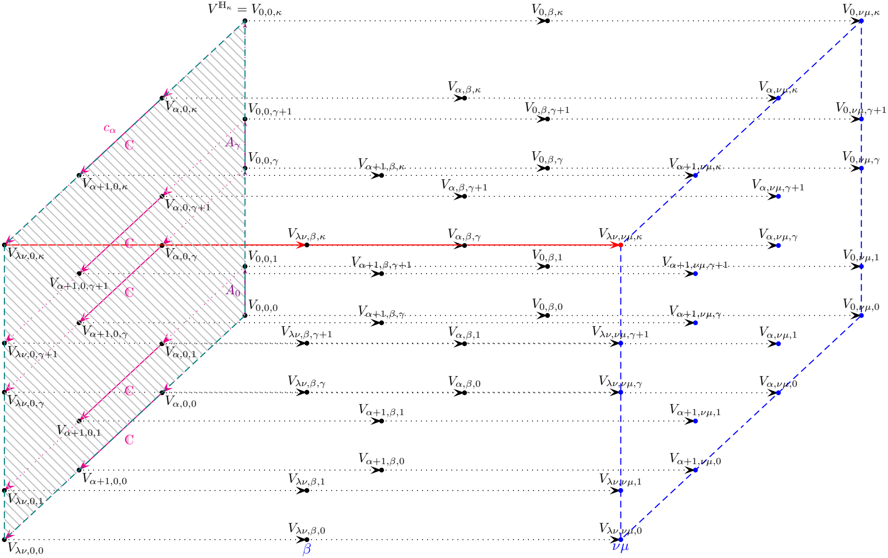
$V_{\alpha,0,0}$
$V_{\alpha+1,\beta,1}$
$V_{\alpha+1,\nu\mu,1}$
$V_{\alpha+1,0,1}$
$V_{\alpha+1,\beta,0}$
$V_{\alpha+1,\nu\mu,0}$
$V_{\alpha+1,0,0}$
$V_{\lambda\nu,\beta,1}$
$V_{\lambda\nu,\nu\mu,1}$
$V_{\lambda\nu,0,1}$
$V_{\lambda\nu,\beta,0}$
$V_{\lambda\nu,\nu\mu,0}$
$V_{\lambda\nu,0,0}$
β
$\nu\mu$


$V^{\mathbb{H}_\kappa} = V_{0,0,\kappa}$
$V_{0,\beta,\kappa}$
$V_{0,\nu\mu,\kappa}$
$V_{\alpha,\beta,\kappa}$
$V_{\alpha,\nu\mu,\kappa}$
$V_{\alpha,0,\kappa}$
$V_{0,0,\gamma+1}$
$V_{0,\beta,\gamma+1}$
$V_{0,\nu\mu,\gamma+1}$
c_α
$\mathbb{C}$
A_γ
$V_{0,0,\gamma}$
$V_{\alpha+1,\beta,\kappa}$
$V_{0,\beta,\gamma}$
$V_{\alpha+1,\nu\mu,\kappa}$
$V_{0,\nu\mu,\gamma}$
$V_{\alpha+1,0,\kappa}$
$V_{\alpha,\beta,\gamma+1}$
$V_{\alpha,\nu\mu,\gamma+1}$
$V_{\alpha,0,\gamma+1}$
$V_{\lambda\nu,\beta,\kappa}$
$V_{\alpha,\beta,\gamma}$
$V_{\lambda\nu,\nu\mu,\kappa}$
$V_{\alpha,\nu\mu,\gamma}$
$V_{\lambda\nu,0,\kappa}$
$V_{\alpha,0,\gamma}$
$V_{0,0,1}$
$V_{\alpha+1,\beta,\gamma+1}$
$V_{0,\beta,1}$
$V_{\alpha+1,\nu\mu,\gamma+1}$
$V_{0,\nu\mu,1}$
$V_{\alpha+1,0,\gamma+1}$
A_0
$V_{0,0,0}$
$V_{\alpha+1,\beta,\gamma}$
$V_{0,\beta,0}$
$V_{\alpha+1,\nu\mu,\gamma}$
$V_{0,\nu\mu,0}$
$V_{\alpha+1,0,\gamma}$
$V_{\lambda\nu,\beta,\gamma+1}$
$V_{\alpha,\beta,1}$
$V_{\lambda\nu,\nu\mu,\gamma+1}$
$V_{\alpha,\nu\mu,1}$
$V_{\lambda\nu,0,\gamma+1}$
$V_{\alpha,0,1}$
$V_{\lambda\nu,\beta,\gamma}$
$V_{\alpha,\beta,0}$
$V_{\lambda\nu,\nu\mu,\gamma}$
$V_{\alpha,\nu\mu,0}$
$V_{\lambda\nu,0,\gamma}$
$V_{\alpha,0,0}$
$V_{\alpha+1,\beta,1}$
$V_{\alpha+1,\nu\mu,1}$
$V_{\alpha+1,0,1}$
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$V_{\alpha+1,\nu\mu,0}$
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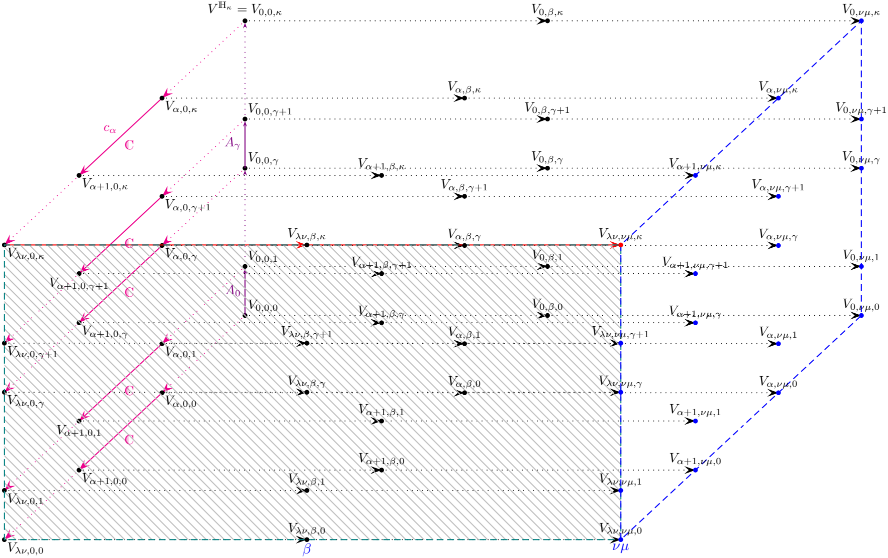
$V_{\lambda\nu,\beta,0}$
$V_{\lambda\nu,\nu\mu,0}$
$V_{\lambda\nu,0,0}$
β
$\nu\mu$


$V^{\mathbb{H}_\kappa} = V_{0,0,\kappa}$
$V_{0,\beta,\kappa}$
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$V_{\alpha,\beta,\kappa}$
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$V_{0,\nu\mu,\gamma}$
$V_{\alpha+1,0,\kappa}$
$V_{\alpha,\beta,\gamma+1}$
$V_{\alpha,\nu\mu,\gamma+1}$
$V_{\alpha,0,\gamma+1}$
$V_{\lambda\nu,\beta,\kappa}$
$V_{\alpha,\beta,\gamma}$
$V_{\lambda\nu,\nu\mu,\kappa}$
$V_{\alpha,\nu\mu,\gamma}$
$V_{\lambda\nu,0,\kappa}$
$V_{\alpha,0,\gamma}$
$V_{0,0,1}$
$V_{\alpha+1,\beta,\gamma+1}$
$V_{0,\beta,1}$
$V_{\alpha+1,\nu\mu,\gamma+1}$
$V_{0,\nu\mu,1}$
$V_{\alpha+1,0,\gamma+1}$
A_0
$V_{0,0,0}$
$V_{\alpha+1,\beta,\gamma}$
$V_{0,\beta,0}$
$V_{\alpha+1,\nu\mu,\gamma}$
$V_{0,\nu\mu,0}$
$V_{\alpha+1,0,\gamma}$
$V_{\lambda\nu,\beta,\gamma+1}$
$V_{\alpha,\beta,1}$
$V_{\lambda\nu,\nu\mu,\gamma+1}$
$V_{\alpha,\nu\mu,1}$
$V_{\lambda\nu,0,\gamma+1}$
$V_{\alpha,0,1}$
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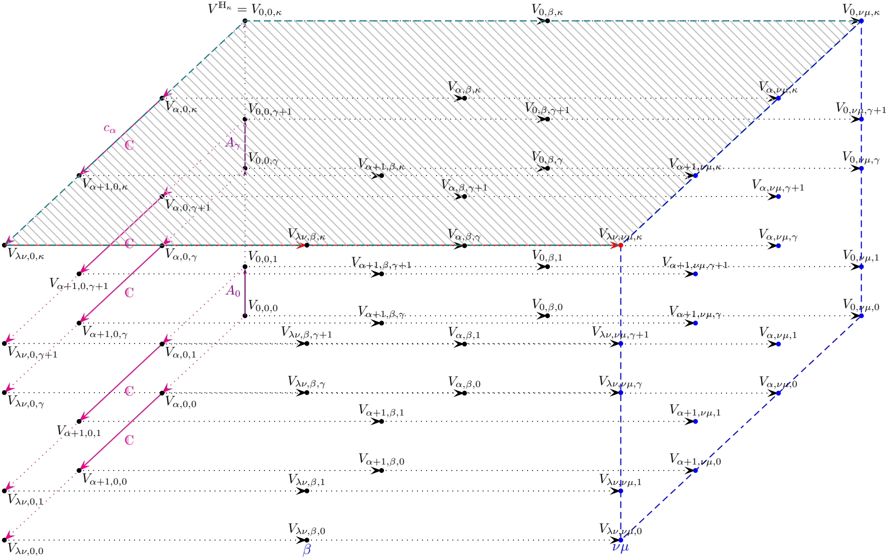
$V_{\lambda\nu,0,1}$
$V_{\lambda\nu,\beta,0}$
$V_{\lambda\nu,\nu\mu,0}$
$V_{\lambda\nu,0,0}$
β
$\nu\mu$



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$V_{\alpha+1,\nu\mu,\gamma+1}$
$V_{0,\nu\mu,1}$
$V_{\alpha+1,0,\gamma+1}$
A_0
$V_{0,0,0}$
$V_{\alpha+1,\beta,\gamma}$
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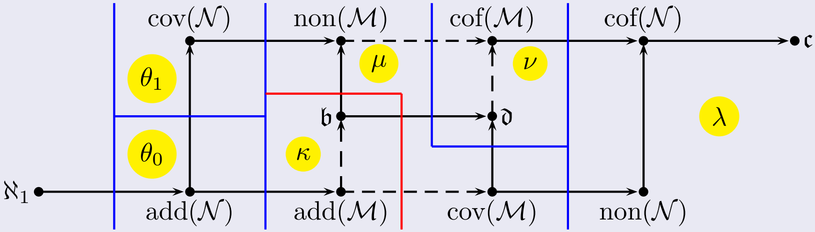
$V_{\lambda\nu,\beta,0}$
$V_{\lambda\nu,\nu\mu,0}$
$V_{\lambda\nu,0,0}$
β
$\nu\mu$

Theorem

Let $\theta_0 \leq \theta_1 \leq \kappa \leq \mu \leq \nu$ *be uncountable regular cardinals,* $\nu \leq \lambda$ *such that* $\lambda^{<\theta_1} = \lambda$*. Then, there is a ccc poset forcing*

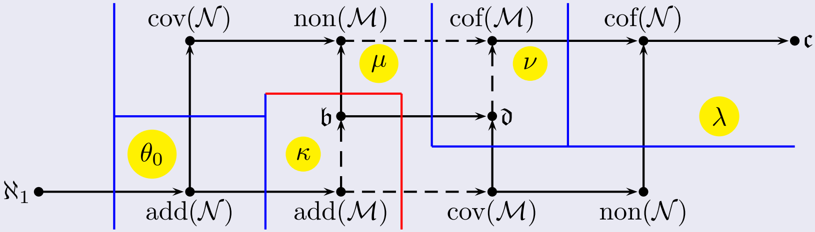


and $\mathfrak{a} = \mathfrak{b}$.

More examples

Theorem

Let $\theta_0 \leq \kappa \leq \mu \leq \nu$ *be uncountable regular cardinals,* $\nu \leq \lambda$ *such that* $\lambda^{<\theta_0} = \lambda$*. Then, there is a ccc poset forcing*



and $\mathfrak{a} = \mathfrak{b}$.

Questions

Question

What happens to $\mathfrak{a}$ in a FS-iteration of $\mathbb{D}$ of lenght $\mu > \aleph_1$ (regular) over a ground model where $\mathfrak{c} > \mu$?

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What happens to $\mathfrak{a}$ in a FS-iteration of $\mathbb{D}$ of lenght $\mu > \aleph_1$ (regular) over a ground model where $\mathfrak{c} > \mu$?

Question

Is it possible to obtain $\mathfrak{a} = \mathfrak{b}$ in **Goldstern-M.-Shelah (2016)** model of