# ON NIKODYM'S UNIFORM BOUNDEDNESS PRINCIPLE

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An infinite Boolean algebra $\mathcal{A}$ is said to have *the Nikodym property* provided that every sequence of measures $\langle \mu_n : n \in \omega \rangle$ on $\mathcal{A}$ which is pointwise bounded (ie. $\sup_n |\mu_n a| < \infty$ for every $a \in \mathcal{A}$) is uniformly bounded (ie. $\sup_n \|\mu_n\| < \infty$). It is known that such an algebra cannot have cardinality less than $\max(\mathfrak{b}, \mathfrak{s})$ where $\mathfrak{b}$ and $\mathfrak{s}$ are respectively the bounding number and the splitting number. However, all the known so far examples of algebras with the Nikodym property have cardinality not less than the continuum $\mathfrak{c}$. Assuming that the cofinality of the Lebesgue null ideal is $\omega_1$, I will provide a brief sketch of an argument leading to the construction of a Boolean algebra with the Nikodym property and cardinality $\omega_1$. This gives a new example of an algebra with cofinality $\omega_1$ as well as contributes a new consistent counterexample to the Efimov problem.

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