Ramsey classes of trees

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Outline

preliminaries

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- we assume M is sufficiently saturated, so if a small set exists by compactness in an elementary extension of M, it exists in M..
- We wish to study the theory of M.

types

- Recall the **type** of an element,

  $\mathrm{tp}^L(\overline{a}; M) = \{\varphi(\overline{x}) \text{ an } L\text{-formula } \mid M \models \varphi(\overline{a})\}$

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- We also have the notion of **quantifier-free type**,

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- Roman letters signify the underlying set of a structure, e.g. $\mathcal{O}$ has underlying set O, $\mathcal{I}$ has underlying set I ...

order-indiscernible sets

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Definition

$B = \{b_i \mid i \in O\}$ is an **order-indiscernible set** if for all $n \geq 1$, for all $i_1, \ldots, i_n$, $j_1, \ldots, j_n$ from O,

$(i_1, \ldots, i_n) \mapsto (j_1, \ldots, j_n)$ is an order-isomorphism $\Rightarrow$

$$\text{tp}^L(b_{i_1}, \ldots, b_{i_n}; M) = \text{tp}^L(b_{j_1}, \ldots, b_{j_n}; M)$$

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Definition ([She90])

$B = \{b_i : i \in I\}$ is an **$\mathcal{I}$-indexed indiscernible set** if for all $n \geq 1$, for all $i_1, \ldots, i_n$, $j_1, \ldots, j_n$ from I,
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- Say that B is Δ-$\mathcal{I}$-indexed indiscernible for $\Delta \subseteq L$ if we replace L in the definition by Δ.

background: classification theory

- A theory T is **stable** if it does not have the order property, i.e., there is no formula $\varphi(\overline{x};\overline{y})$ in the language of T and parameters $\{\overline{a}_i\}_{i<\omega}$ from some model of T such that

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- Equivalently, for any definable set $X \subset M^n$ (using parameters from the ambient model), $X \cap A^n$ is definable using only parameters from A – the trace of a definable set on A is A-definable.

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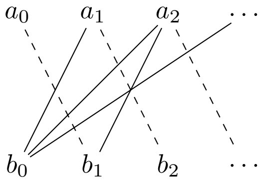

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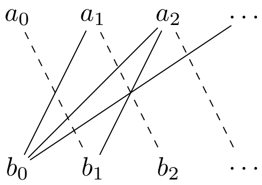

Get θ s.t. $\theta((a_i, b_i), (a_j, b_j)) \Leftrightarrow i < j$.

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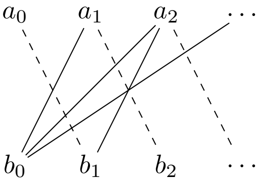

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- How do we know there is no order in 1-variable?

using Ramsey's theorem

- Suppose for contradiction there is $\varphi(x,y)$ such that $\ell(x) = \ell(y) = 1$ and parameters $A = \{a_i\}_i$ with

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- By Ramsey's theorem, there is an indiscernible sequence $B = \{b_i\}_i$ with

$$i < j \Rightarrow M \models \varphi(b_i, b_j)$$

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  (all "increasing pairs" (i, j) are colored "$M \models \varphi(a_i, a_j)$" – find a large enough homogeneous subset $A_0 \subset A$ to stand for a fragment of B)

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- By indiscernibility, B is a complete graph or an empty graph (thus an indiscernible set) contradicting disagreement on $\varphi(x, y)$.

NIP *

Definition

A theory T has NIP ("not the Independence property") if there is no formula $\varphi(\overline{x};\overline{y})$ in the language of T and parameters $\{\overline{a}_i\}_{i<\omega}$ from some model of T such that

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- The theory of $(\mathbb{Q},<)$ is NIP.
- The theory of $(\mathbb{Z},+,\cdot)$ is not NIP.

characterization theorems *

Theorem (Shelah)

A theory T is stable iff any infinite order-indiscernible sequence in a model of T is an indiscernible set.

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- Let $\mathcal{I}$ be any graph with an ordering on its vertices (in signature $\{<, E\}$) that contains a copy of every finite ordered graph.

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Theorem

A theory T is NIP iff any $\mathcal{I}$-indexed indiscernible set in a model of T is an order-indiscernible set.

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Theorem

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- Can't do better because of $\mathrm{Th}((\mathbb{Q}, <))$.

EM-types *

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Definition

$$\mathrm{EM}(A) = \{\varphi(x_{i_1}, \ldots, x_{i_n}) \mid \text{ for all } (j_1, \ldots, j_n) \sim (i_1, \ldots, i_n), \; M \models \varphi(a_{j_1}, \ldots, a_{j_n})\}$$

examples

- The EM-type encodes rules such as

$$q(i_1, \ldots, i_n) \Rightarrow \; M \vDash \varphi(a_{i_1}, \ldots, a_{i_n})$$

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Example

Consider a set $A = \{a_i \mid i \in (\omega, <)\}$ such that $i < j \Rightarrow \varphi(a_i, a_j)$ but $\neg\varphi(a_1, a_0)$ and $\varphi(a_2, a_0)$, then

$$\varphi(x_0, x_1), \varphi(x_0, x_2), \varphi(x_1, x_2) \ldots \in \mathrm{EM}(A)$$

but

$$\varphi(x_i, x_j), \neg\varphi(x_i, x_j) \notin \mathrm{EM}(A), \text{ for } i > j$$

the modeling property

Definition

$\mathcal{I}$-indexed indiscernible sets have the modeling property if for all I-indexed parameters $A = \{a_i : i \in I\}$ *in any structure* M, there exists an $\mathcal{I}$-indexed indiscernible set B s.t.

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- For which $\mathcal{I}$ do $\mathcal{I}$-indexed indiscernible sets have the modeling property?

dictionary theorem

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Theorem ([Sco13])

For $\mathcal{I}$ as above, $\mathcal{I}$-indexed indiscernible sets have the modeling property just in case age($\mathcal{I}$) is a Ramsey class.

application

- $\mathcal{I}_0 = (\omega^{<\omega}, \trianglelefteq, \wedge, <_{\text{lex}})$

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Theorem (Takeuchi-Tsuboi)

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Corollary (Leeb)

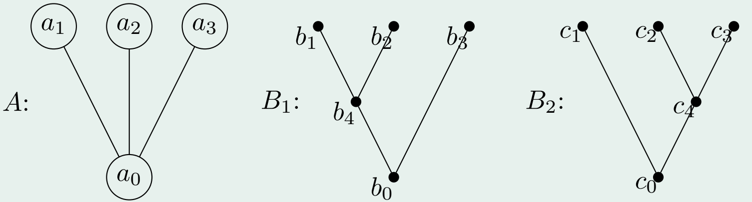
age($\mathcal{I}_0$) is a Ramsey class.

- Removing $\wedge$ destroys the Ramsey property.

$\mathcal{K} = \mathrm{age}(\mathcal{I}_0 \restriction \{\trianglelefteq, <_{\mathrm{lex}}\})$ is not a Ramsey class *

Proof.

By [Neš05], if $\mathcal{K}$ is a Ramsey class, then $\mathcal{K}$ has the amalgamation property. However, an example analyzed in Takeuchi-Tsuboi provides a counterexample to amalgamation. Consider embeddings $a_i \mapsto b_i, c_i$.



□

trees *

- $\mathcal{I}_s = (\omega^{<\omega}, \trianglelefteq, \wedge, <_{\text{lex}}, (P_n)_{n<\omega})$

  where $\trianglelefteq$ is the partial tree-order, $\wedge$ is the meet function in this order, $<_{\text{lex}}$ is the lexicographical order, and the P_n are predicates picking out the n-th level of the tree

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finitary infinitary

Theorem ([She90])

For every $n, m < \omega$ *there is some* $k = k(n, m) < \omega$ *such that for any infinite cardinal* χ, *the following is true of* $\lambda := \beth_k(\chi)^+$: *for every* $f : ({}^{n\geq}\lambda)^m \to \chi$ *there is an* L_s*-subtree* $I \subseteq {}^{n\geq}\lambda$ *such that*

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(i) $\langle\rangle \in I$ *and whenever* $\eta \in I \cap {}^{n>}\lambda$, $||\{\alpha < \lambda : \eta^\frown \langle\alpha\rangle \in I\}|| \geq \chi^+$.

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For every $n, m < \omega$ there is some $k = k(n,m) < \omega$ such that for any infinite cardinal χ, the following is true of $\lambda := \beth_k(\chi)^+$: for every $f : ({}^{n\geq}\lambda)^m \to \chi$ there is an L_s-subtree $I \subseteq {}^{n\geq}\lambda$ such that

(i) $\langle\rangle \in I$ *and whenever* $\eta \in I \cap {}^{n>}\lambda$, $||\{\alpha < \lambda : \eta^\frown\langle\alpha\rangle \in I\}|| \geq \chi^+$.

(ii)$_f$ *If* $\bar{\eta}, \bar{\nu} \in I$ *are such that* $\bar{\eta} \sim_{\mathcal{I}_s} \bar{\nu}$ *then* $f(\eta_0, \ldots, \eta_{m-1}) = f(\nu_0, \ldots, \nu_{m-1})$.

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Both yield that $\mathcal{I}_s$-indexed indiscernibles have the modeling property, the second by way of the dictionary theorem.

Thanks

Thanks for your attention!

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