Algebraic characterisation of pseudo-elementary and second-order classes

János Balázs Ivanyos

Supervisor: Gábor Sági

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  - Let us find closure conditions $\leftrightsquigarrow$ algebraic characterization.

HSP Theorem (Birkhoff '35)

A class of algebraic structures is axiomatizable by equations if and only if it is closed under **H**, **S**, and **P**.

Axiomatizability in first-order logic

Theorem (Keisler-Shelah '71)

Let $\mathcal{A}$ and $\mathcal{B}$ be two τ-structures. Then the following are equivalent:

- $\mathcal{A} \equiv_{FO} \mathcal{B}$;
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Corollary

The class of all finite τ-structures is not definable in first-order logic.

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- Second-order classes: finite τ-structures, graphs that are 3-colorable.
- First-order tools do not work: ultraproducts do not preserve the truth of second-order formulas, so Łoś' lemma does not hold in second-order logic; moreover, the compactness theorem also fails.

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Inseparability $\approx$ "there is a nice isomorphism between their ultrapowers".

Inseparability baby version

- Fix an ultraproduct $\mathcal{C} = \prod_{i \in I} \mathcal{C}_i / \mathcal{H}$. Then for $0 < k$ a relation $R \subseteq {}^k C$ is defined to be decomposable in the ultraproduct, iff for every $i \in I$ there exists a relation $R_i \subseteq {}^k C_i$ such that $R = \prod_{i \in I} R_i / \mathcal{H}$.

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- $\{\mathcal{B}\}$ is defined to be *inseparable* from $\{\mathcal{A}\}$, if there exist ultrapowers $\mathcal{A}' = {}^J \mathcal{A} / \mathcal{F}$, $\mathcal{B}' = {}^L \mathcal{B} / \mathcal{G}$ and an isomorphism

$$f : \mathcal{A}' \to \mathcal{B}'$$

such that, if R is a relation of $\mathcal{A}$, and $S = {}^J R / \mathcal{F}$, then $f^*(S)$ is decomposable in $\mathcal{B}'$.

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(2) K is axiomatizable by a **finite** set of second-order formulas iff K is closed under complete inseparability.

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- Since the 1960s it was an open problem to give a purely algebraic characterization.

PC classes

- $K \in PC$ iff there exists a $\psi \in \Sigma^1_1$ ($\psi \equiv \exists R_0, \ldots, R_{n-1} \phi$ where ϕ is first-order) formula such that $K = \mathrm{Mod}(\psi)$.

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Theorem (Ivanyos)

The following are equivalent:

(1) $K \in \mathrm{PC}$;

(2) K is axiomatizable by a finite set of existential second-order formulas;

(3) K is strongly closed under inseparability;

(4) K is closed under ultraproducts and inseparability.

Connection to NP

Theorem (Fagin '74)

The class K of finite structures is in NP exactly when there exists a $\phi \in \Sigma^1_1$ formula such that $K = \mathrm{Mod}_{<\omega}(\phi)$.

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Theorem (Ivanyos)

Let K be a class of finite τ-structures. Then the following are equivalent:

(1) $K \in \mathrm{NP}$;

(2) For every $\{\mathcal{A}_i : i \in I\} \subseteq K$, if $\{\mathcal{B}_j : j \in J\}$ is a set of finite τ-structures that is inseparable from $\mathcal{A}$, then $K \cap \{\mathcal{B}_j : j \in J\} \neq \emptyset$.

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$$C(\Gamma) = \left\{ x \in 2^\kappa : \begin{array}{l} \text{there is a model } \mathcal{A}, \text{ such that} \\ \gamma_i \in Th_\Gamma(\mathcal{A}) \text{ if and only if } x(i) = 1 \end{array} \right\}.$$

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- Every $x \in C(\Gamma)$ is the characteristic function of $Th_\Gamma(\mathcal{A})$, for some $\mathcal{A}$ τ-structure.
- Consider the subspace topology on $C(\Gamma)$ inherited from the κ-Cantor set 2^κ.

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- Hence, in order to characterize the Γ-definable classes, we have to understand the topology of the space $C(\Gamma)$.

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- So, if $\Gamma \nsubseteq FO$ we need to find an operation that induces a closure operation on the space $C(\Gamma)$.
- In second-order, this operation will be the inseparability (its variants).

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Theorem ZFC (Ivanyos)

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(2) K^c has property (A) and K has property (B).

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Thank you for your attention!