

Weakly Compact Cardinals in the Bristol Model

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Abstract

This talk is based on joint work with Asaf Karagila. Without assuming the axiom of choice, many definitions of large cardinals which are equivalent over models of \mathbf{ZFC} can be separated. We will focus on defining weak compactness using the tree property, and ask whether this definition can be separated from other definitions of weakly compact cardinals without choice.

To generate models of $\neg\mathbf{AC}$, we will often use symmetric extensions. However, this method can only generate ‘small’ failures of choice, specifically models of \mathbf{SVC} (small violations of choice). By instead using a class-length iteration of symmetric extensions, we can instead work in the Bristol Model, which exhibits a ‘deeper’ failure of choice.

I will give a brief introduction to symmetric extensions and the Bristol model, then overview current research on how weakly compact cardinals behave in the Bristol Model.

References

- [1] Asaf Karagila. The bristol model: An abyss called a cohen real. *Journal of Mathematical Logic*, 18(02):1850008, November 2018.
- [2] Asaf Karagila. Approaching a bristol model, 2025.