THE RUDIN–BLASS ORDERING OF MEASURES

Abstract. Presented study arose as an effort to lift the rich, set-theoretic results around ultrafilters to the realm of (finitely additive) measures. One of the most fruitful approaches to study the structure of ultrafilters on ω is through Rudin–Keisler and Rudin–Blass orderings. We extend these notions to measures and propose a generalization of the notions of a Q-point and a selective ultrafilter: Q-measures and selective measures. Many symmetries between Q-points and Q-measures hold, however we were also able to find instances where they break. In particular we present an example of a measure which is minimal in the sense of Rudin–Blass but which is not a Q-measure. We also explore the relation between existence of measures and ultrafilters of certain types. Of a particular interest is the fact that the existence of a non-atomic Q-measure is closely tied to the number of Q-points and selective ultrafilters in the model.

The presented results are a joint work with Piotr Borodulin-Nadzieja, Arturo Martinez-Celis and Jadwiga Świerczyńska. See arxiv.org/abs/2504.14678.

Adam Morawski
PhD student at Charles University
& The Czech Academy of Sciences, Prague