

Changing the Length of the Borel Hierarchy

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The structural study of the Borel hierarchy on topological spaces is a foundational goal of descriptive set theory. For a subspace $X \subseteq {}^\omega\omega$ of the reals, we study the length of this hierarchy on X , i.e. the least ordinal α such that every Borel subset of X is Σ^0_α . This can be thought of as a measure of the topological complexity of X .

The topic of this talk is a framework for the surgical alteration of the Borel hierarchy on subspaces of the reals, pioneered by A. Miller. We will discuss Miller's notion of α -forcing, which allows for either collapsing or increasing the length of the Borel hierarchy, and sketch a connection between certain combinatorial properties of the forcing and topological properties of spaces in the generic extension via the framework of ranked forcing. Lastly, some recent developments in this area are presented, such as an extension of the framework to the field of generalized descriptive set theory of an uncountable cardinal $\kappa = \kappa^{<\kappa}$.