Ideal zoo in the Baire space 2

Marcin Michalski

During the talk we will take a look at the translations into the Baire space of several well-known σ-ideals and families originally defined on the Cantor space, using their combinatorial characterizations (see e.g. [1]). These include the ideals of null sets, small sets, those generated by closed measure-zero sets, and the meager sets, leading to their "fake" analogues in the Baire space. In the cases of families related to null sets we will parametrize them with reasonable functions from ω^ω. Namely

Definition 1 *$F \in \mathcal{M}_-$ if there are $x_F \in \omega^\omega$ and a partition of ω into intervals $(I_n)_{n\in\omega}$ such that*

$$F \subseteq \{x \in \omega^\omega : (\forall^\infty n)(x \restriction I_n \neq x_F \restriction I_n)\}.$$

Definition 2 *Let $h \in \omega^\omega$, $\limsup_n h(n) = \infty$ and $F \subseteq \omega^\omega$. We will say that*

- *$F \in f\mathcal{N}(h)$ if there is a sequence $(S_n)_{n\in\omega}$, $S_n \subseteq \omega^n$, $\sum_{n\in\omega} \frac{|S_n|}{h(n)} < \infty$, such that*
$$F \subseteq \{x \in \omega^\omega : (\exists^\infty n)(x \restriction n \in S_n)\};$$
- *$F \in f\mathcal{S}(h)$ if there is a partition of ω into intervals $(I_n)_{n\in\omega}$ and a sequence $(J_n)_{n\in\omega}$, $J_n \subseteq \omega^{I_n}$, $\sum \frac{|J_n|}{h(|I_n|)} < \infty$ such that*
$$F \subseteq \{x \in \omega^\omega : (\exists^\infty n)(x \restriction I_n \in J_n)\};$$
- *$F \in f\mathcal{E}(h)$ if there is a partition of ω into intervals $(I_n)_{n\in\omega}$ and a sequence $(J_n)_{n\in\omega}$, $J_n \subseteq \omega^{I_n}$, $\sum \frac{|J_n|}{h(|I_n|)} < \infty$ such that*
$$F \subseteq \{x \in \omega^\omega : (\forall^\infty n)(x \restriction I_n \in J_n)\};$$

While in the part 1 Łukasz is focusing on individual families and their properties, in the sequel we will deal with the interactions between them. In particular we will try to compare them (with $\subseteq$), find large chains and antichains of ideals $f\mathcal{N}(h)$ for different h, and examine the orthogonality of $f\mathcal{N}(h)$ and $f\mathcal{S}(h)$ to $\mathcal{M}_-$.

These results were obtained together with Łukasz Mazurkiewicz and Szymon Żeberski - see [2].

References

[1] Bartoszyński T., Judah H., *Set theory. On the structure of the real line*, 1st edition, A K Peters, Wellesley, Massachusetts, 1995.

[2] Mazurkiewicz Ł., Michalski M., Żeberski Sz., *An Ideal Zoo in the Baire Space*, arXiv: 2510.27435.