c_0-products of function spaces

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For a Tichonoff topological space X, let $C_p(X)$ denote the space of real continuous functions on X equipped with the pointwise convergence topology. Recall that a topological space X is called pseudocompact if every real continuous function on X is bounded. For a pseudocompact space X, we consider the *sup* norm $\| \ \|$ on the space $C_p(X)$. The c_0-product of spaces $C_p(X)$ is defined as follows

$$\Big(\prod_{n\in\mathbb{N}} C_p(X)\Big)_0 = \Big\{(f_n)_{n\in\mathbb{N}} \in \prod_{n\in\mathbb{N}} X_n : \|f_n\| \to 0\Big\},$$

endowed with the product topology. If the space X is a single point, then the c_0-product above becomes the well known c_0 space of real sequences converging to zero, endowed with the product topology. Gul'ko and Khmyleva proved that c_0 is homeomorphic to its own countable power [GK]. We generalize this theorem and show that, for every pseudocompact space X, the c_0-product $\Big(\prod_{n\in\mathbb{N}} C_p(X)\Big)_0$ is homeomorphic to its own countable power.

References

[GK] S. P. Gul'ko and T. E. Khmyleva *Compactness is not preserved by the t-equivalence relation*, Mathematical Notes of the Academy of Sciences of the USSR, 39, (1986), 484—488.