Separating Regularity Properties with the Raisonnier Filter

Yurii Khomskii

In 1984, Shelah proved the following statement: if all $\mathbf{\Sigma}^1_3$ sets of reals are Lebesgue-measurable, then there is an inaccessible cardinal in L. The proof was simplified (and improved) by Raisonnier, using a construction now called the *Raisonnier Filter*. This filter has complexity $\mathbf{\Sigma}^1_3$ (when defined over L as the ground model), and if the reals of $L[a]$ have size $\aleph_1$ for some parameter a, and if all $\mathbf{\Sigma}^1_2$ sets are measurable, then it is a *rapid filter* — which, in turn, implies that it is non-measurable by a result of Mokobodzki.

I will show how the Raisonnier filter can be used to obtain interesting separation results for regularity properties at the levels $\mathbf{\Delta}^1_3$, $\mathbf{\Sigma}^1_3$, $\mathbf{\Delta}^1_4$ and $\mathbf{\Sigma}^1_4$. For example, consistently with ZFC the statement "all $\mathbf{\Delta}^1_3$ sets are Ramsey" is not equivalent to "all $\mathbf{\Sigma}^1_3$ sets are Ramsey", unlike the corresponding statements for $\mathbf{\Delta}^1_2$ and $\mathbf{\Sigma}^1_2$ sets which are provably equivalent by a result of Judah-Shelah.

Most of this talk is based on older work with Vera Fischer and Sy Friedman, and some unpublished work with Joerg Brendle.