Generalized Ważewski dendrites, generic subcontinua, and generic chains

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We say that a continuum X is a hereditarily equivalent continuum if every non-degenerate subcontinuum of it is homeomorphic to X. This concept is one of the main motivations behind the construction of the pseudo-arc. If considered in the hyperspace of continua of X, denoted by $\mathrm{Cont}(X)$, it means that

$$\mathrm{Cont}(X) \setminus \mathrm{Fin}(X) = \{K \in \mathrm{Cont}(X) \mid K \simeq X\}.$$

This is an open and dense set, hence comeager, thus we can say that the generic subcontinua of X is homeomorphic to X. Therefore, it is natural to ask if there exist other spaces that satisfy this weaker property of having such collection of homeomorphic sets being comeager. We call these spaces generically hereditarily equivalent continua and have shown that the generalized Wazewski dendrites W_M for $M \subseteq \{3, 4, \ldots, \infty\}$ are such spaces. Moreover, in the hyperspace of maximal order arcs of W_M - $\mathrm{MOA}(W_M)$, the chains having every non-degenerate element homeomorphic to W_M make a comeager subset of the maximal order arcs.

Additionally, we consider the action of the homeomorphism group $\mathrm{Homeo}(W_M)$ on $\mathrm{Cont}(X)$ and on $\mathrm{MOA}(W_M)$ and describe a comeager orbit for each of it.

References

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