FAKE VS REAL NULL SETS

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This is an ongoing project with M. Michalski, R. Rałowski and Sz. Żeberski.

The idea comes from the following characterization, which can be found in [1, Lemma 2.5.1]:

Lemma 1. *Suppose that $F \subseteq 2^\omega$ has measure zero. Then there exists a sequence $(S_n)_{n\in\omega}$, $S_n \subseteq 2^n$, such that $\sum_{n\in\omega} \frac{S_n}{2^n} < \infty$ and*

$$F \subseteq \{x \in 2^\omega : (\exists^\infty n \in \omega)(x{\restriction}n \in S_n)\}.$$

In [2] we rewrite the above characterization directly in a context of the Baire space to obtain a natural example of an ideal resembling measure. However, the condition that limits the size of S_n s does not seem natural in the Baire space. Therefore, we would like to parametrize it with a function $h \in \omega^\omega$, introducing "fake-null" sets.

Definition 2. *Let $h \in \omega^\omega$, $\limsup_n h(n) = \infty$. We say that $F \in f\mathcal{N}(h)$ if there is a sequence $(S_n)_{n\in\omega}$, $S_n \subseteq \omega^n$, $\sum \frac{|S_n|}{h(n)} < \infty$, satisfying a condition*

$$F \subseteq \{x \in \omega^\omega : (\exists^\infty n)(x{\restriction}n \in S_n)\}.$$

Moreover, we will define fake-small and fake-$\mathcal{E}$ sets. We will explore properties of those objects (e.g. being and ideal) and check whether known facts from their "real" versions hold (e.g. is every fake-null set a sum of two fake-smalls?).

References

[1] T. Bartoszyński and H. Judah. *Set theory: On the structure of the real line.* A K Peters. Ltd., 1995.

[2] Łukasz Mazurkiewicz, Marcin Michalski, Robert Rałowski, and Szymon Żeberski. "On algebraic sums, trees and ideals in the Baire space". In: *arXiv:2409.17748* (2024).

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