Scales and combinatorial covering properties

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A $\mathfrak{b}$-*scale set* is a subset of $\mathrm{P}(\omega)$ of the form $\{x_\alpha \colon \alpha < \mathfrak{b}\} \cup \mathrm{Fin}$, where $\{x_\alpha \colon \alpha < \mathfrak{b}\}$ is an unbounded set in $[\omega]^\omega$ and for all $\alpha < \beta < \mathfrak{b}$ we have $x_\alpha \leq^* x_\beta$. These sets play a crucial role in the investigation of combinatorial covering properties. Bartoszyński and Shelah showed that each $\mathfrak{b}$-scale set is Hurewicz but not σ-compact which is a counterexample in ZFC for Hurewicz's conjecture. Under additional set-theoretical assumptions, by the results of Bartoszyński, Tsaban and Weiss all finite powers of a $\mathfrak{b}$-scale set are Rothberger and Hurewicz.

Recently, $\mathfrak{b}$-scale sets and their generalizations using filters were intensively investigated in products with spaces having Hurewicz, Scheepers or Menger covering properties. Thus far, another classical properties from the second row of the Scheepers Diagram have not been considered in this context. We present new results in this field.