PRODUCTIVITY OF SELECTIVE COVERING PROPERTIES

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A topological space X is *Hurewicz* if for any sequence $\mathcal{U}_0, \mathcal{U}_1, \ldots$ of open covers of X, there are finite families $\mathcal{F}_0 \subseteq \mathcal{U}_0, \mathcal{F}_1 \subseteq \mathcal{U}_1, \ldots$ such that the family $\{\bigcup \mathcal{F}_n : n \in \omega\}$ is a γ-cover of X, i.e., the sets $\{n : x \notin \bigcup \mathcal{F}_n\}$ are finite for all points $x \in X$. In a similar way we also define another covering properties as Scheepers and Menger. The following implications hold

$$\text{Hurewicz} \longrightarrow \text{Scheepers} \longrightarrow \text{Menger}$$

and the first one is not reversible in ZFC. For a topological property P, a set of reals X is *productively P*, if for any set of reals Y with the property P, the product space $X \times Y$ also has the property P. Szewczak and Tsaban proved that if $\mathfrak{b} = \mathfrak{d}$, then every productively Menger set is productively Hurewicz. We develop their methods and consider relations between productively Scheepers and productively Hurewicz sets.

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