IDEALS ON ω AND THE NIKODYM VS THE GROTHENDIECK PROPERTY OF BOOLEAN ALGEBRAS

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An infinite Boolean algebra $\mathcal{A}$ is said to have the Nikodym property when every sequence of measures $\langle \mu_n : n \in \omega \rangle$ on $\mathcal{A}$ such that $\mu_n(A) \to 0$ for all $A \in \mathcal{A}$ is uniformly bounded. For a free filter F on ω we consider the space $N_F = \omega \cup \{p_F\}$, where ω is a discrete subspace and open neighborhoods of p_F are of the form $X \cup \{p_F\}$ for $X \in F$.

We define a class $\mathcal{AN}$ of ideals $\mathcal{I}$ on ω such that any Boolean algebra $\mathcal{A}$ cannot have the Nikodym property when $N_{\mathcal{I}^*}$, for some $\mathcal{I} \in \mathcal{AN}$, is homeomorphically embedded into the Stone space $St(\mathcal{A})$ of ultrafilters on $\mathcal{A}$. We characterize $\mathcal{AN}$ in terms of density ideals, and using this we conclude that $\mathcal{I} \in \mathcal{AN}$ if and only if $\mathcal{I}$ is contained in some summable ideal. Moreover, we show connections of the class $\mathcal{AN}$ with the family of totally bounded ideals.

Our results apply to the Grothendieck property of Boolean algebras, which is closely related to the Nikodym property. Using results of Marciszewski and Sobota concerning the Grothendieck property and N_F spaces, we obtain large families of algebras with the Nikodym property but without the Grothendieck property

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