

Hyperfiniteness on topological Ramsey spaces

A central goal in the area of countable Borel equivalence relations (CBERs) is understanding hyperfinite CBERs. So far, roughly speaking, all known proofs for non-hyperfiniteness use measures, while the tool of Baire category is not useful in this context, as all CBERs are hyperfinite on a co-meager invariant set. Regarding topological Ramsey spaces (in the sense of Todorćević), a classical result of Mathias and Soare states that every CBER on the Ellentuck space $[\mathbb{N}]^{\mathbb{N}}$ is hyperfinite on a Ramsey positive set. There are similar results for certain other topological Ramsey spaces: [1] proves the analogous canonization result for the Millikan space, and recently, Wang and Panagiotopoulos showed that CBERs are hyperfinite (in fact, even smooth) on positive sets in the Carlson-Simpson space [2]. We generalize these statements to all topological Ramsey spaces. This is joint work with Zoltán Vidnyánszky.

References

- [1] V. Kanovei, M. Sabok, and J. Zapletal. *Canonical Ramsey theory on Polish spaces*, volume 202. Cambridge University Press, 2013.
- [2] A. Panagiotopoulos and A. Wang. Every CBER is smooth below the Carlson-Simpson generic partition. *arXiv:2206.14224*, 2022.