GENERALIZED TUKEY RELATIONS IN SOLOVAY MODELS

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In the context of ZFC, the cofinalities of ordered sets are extensively studied in the research area of the cardinal invariants. The Tukey reducibility plays a crucial role in comparing the cofinal types of directed sets. However, the basics of the Tukey reducibility depend on the axiom of choice, so it is not a suitable relation in ZF.

In this talk, we introduce a generalized version of the Tukey reducibility, called the pre-Tukey reducibility, and see that it works well in ZF. Specifically, we show that in the Solovay model the cofinal types of the following directed sets are different each other in the sence of the pre-Tukey reducibility.

$$(\omega^\omega, \leq^*), (\mathcal{M}, \subseteq), (\mathcal{N}, \subseteq), (\omega_1, <), ([\omega^\omega]^\omega, \subseteq).$$

This is a joint work with Hiroshi Sakai.