On some applications of Δ-spaces and Δ_1-spaces

ABSTRACT

A (topological) space X has the Δ-property (and then X is called a Δ-space) if any disjoint sequence $\{A_n : n \in \omega\}$ of subsets of X has a point-finite open expansion, i.e., there exists a point-finite family $\{U_n : n \in \omega\}$ of open subsets of X such that $A_n \subset U_n$ for every $n \in \omega$ (Kąkol-Leiderman). They established that a Tychonoff space X is a Δ-space iff $C_p(X)$ is distinguished, so the Δ-property turned out to have nice applications in C_p-theory and Functional Analysis. Kąkol, Kurka and Leiderman showed that the space $C_p(X)$ is κ-Fréchet-Urysohn for any Δ-space X. This result motivated the authors to introduce a natural generalization of Δ-spaces: A space X has the Δ_1-property if any disjoint sequence $\{A_n : n \in \omega\}$ of countable subsets of X has a point-finite open expansion (Kąkol-Kurka-Leiderman). There are quite a few results showing that the Δ-property of X together with some kind of extra natural conditions on X implies the scatteredness of X. For example, a countably compact Δ-space must be scattered, and if X is Cech-complete, then even the Δ_1-property of X implies X is scattered. Therefore, it is natural to search for which classes of spaces X the (hereditary) Baire property of a Δ_1-space X implies that X is scattered or at least has isolated points. Recall that a compact space X is scattered iff X is a Δ_1-space. This line of research (recently developed, among others, by Ferrando, Kąkol, Kurka, Leiderman, Saxon, Szeptycki and Tkachuk) will be discussed during the talk. Applications are provided.